The singularity appearing in this metric at $r = r_g$ is retained in the coefficient accompanying dR^2.

Let us see what results can be obtained from a closed, isotropic Friedman's model. Since $r = a\sin\chi$ and $R = 2a_0\chi$, we easily obtain

$$c\tau_r = \frac{1}{a^{\cdot}\sin\chi}, \qquad c\tau_R = -\frac{r\cos\chi}{2a^{\cdot}a_0\sin^2\chi}$$

Knowing that $r_\chi = a\cos\chi$, we have $r' = r_R = a\,(2a_0)^{-1}\cos\chi$. At the same time from $e^\omega = r'^2 / (1 + f)$ (remembering that $f = -\sin^2\chi$), we find $e^\omega = (a / 2a_0)^2$. Substituting the values for $c\tau_r$, $c\tau_R$ and e^ω into (13) we have

$$-ds^2 = \frac{r\cos\chi}{a^{\cdot 2}a_0\sin^2\chi}\,dr\,dR + \frac{a^2}{4a_0^2}\left(1 - \frac{\cos^2\chi}{a^{\cdot 2}\sin^2\chi}\right)dR^2 - \frac{dr^2}{a^{\cdot 2}\sin^2\chi} + r^2 d\Omega^2$$

which is convenient e.g. for writing out the equations $T_{i,k}^k = 0$ when two quasi-linear equations defining ε and u can be obtained simultaneously.

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ON REGULAR PRECESSIONS OF A HEAVY GYROSTAT

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The regular precessions of a heavy asymmetric gyrostat are found by direct integration of the system of Zhukovskii equations written in the principal axes of inertia. The properties of these motions are investigated; the possibility of controlling them is revealed. Forces capable of causing a regular precession in the gyrostats are investigated.

An idea was developed in [1] on the preferability of investigating the motion of a heavy gyrostat fixed at one point before the investigation of the motions of the classical rigid body (*) (see footnote on the next page).

Still earlier [2] an attempt at such an investigation was made for an asymmetric gyrostat by Grioli's method and the dynamic possibility of regular precessions in such gyrostats was proved. Later on in [4] a solution of this problem was obtained in special nonprincipal axes of inertia, which was not written as a function of time.

1. We consider a heavy gyrostat which has, for example, cavities filled with an ideal incompressible liquid circulating with a gyrostatic moment λ which is constant in absolute value. We assume that on the position of the gyrostat's center of mass and on its principal moments of inertia the constraints

$$x_0\sqrt{B-C} = z_0\sqrt{A-B}, \quad y_0 = 0, \qquad (A-B)(B-C) > 0 \tag{1.1}$$

are imposed. Geometrically they denote the disposition of the gyrostat's center of mass on the perpendicular to one of the circular cross sections of its ellipsoid of inertia for a fixed point. Furthermore, we assume that the gyrostatic moment λ is directed along the barycentric axis, i.e. the axis carrying the gyrostat's center of gravity (Fig. 1). Then, in projections onto the system's moving axes,

$$\lambda = \lambda_1 \mathbf{i} + \lambda_3 \mathbf{k} \qquad \lambda_1 = \lambda_x = \frac{x_0}{l}\lambda, \quad \lambda_2 = \lambda_y = 0, \quad \lambda_3 = \lambda_z = \frac{z_0}{l}\lambda \tag{1.2}$$

Under the constraints (1.1) adopted, with due regard to (1.2) the system of Zhukovskii differential equations are written as

$$\begin{aligned} A\frac{dp}{dt} &= (B-C)qr - \frac{z_0}{l}\lambda q + \mu z_0\gamma_2 \qquad (\mu = Mg) \\ B\frac{dq}{dt} &= -(A-C)pr + \frac{\lambda}{l}(z_0 p - x_0 r) + \mu(x_0\gamma_3 - z_0\gamma_1) \\ C\frac{dr}{dt} &= (A-B)pq + \frac{x_0}{l}\lambda q - \mu x_0\gamma_2 \end{aligned} \tag{1.3}$$

where M is the mass of the body of the carrier and of the liquid. The system of kinematic equations remains as before.

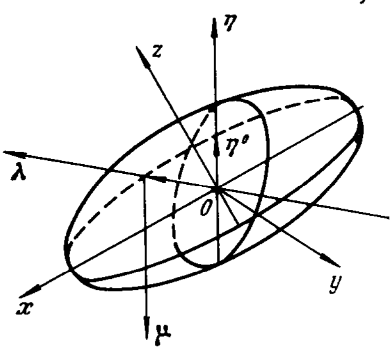

Fig. 1

We multiply the first equation of (1.3) by x_0, the third by z_0, and we add them together. Keeping in mind the equalities obtained from (1.1)

$$A - B = kx_0^2, \; B - C = kz_0^2 \tag{1.4}$$

$$A - C = kl^2, \; E = kx_0z_0 \quad (l = \sqrt{x_0^2 + z_0^2})$$

where k is a constant having the dimension of mass, l is the distance of the gyrostat's center of mass from the fixed point, we obtain

$$\frac{d}{dt}(Ax_0p + Cz_0r) = kx_0z_0(x_0p + z_0r)q \tag{1.5}$$

To find the regular precessions of the heavy gyrostat we should assume [5, 6]

$$(\mathbf{l}\cdot\boldsymbol{\omega}) = x_0p + z_0r = \text{const} = m \tag{1.6}$$

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*) Kharlamova, E. I., Algebraic invariant relations of the differential equations of rigid body dynamics. Doctorate Dissertation, 1971.

Using the result of differentiating (1.6) with respect to time, from Eq. (1.5) we obtain

$$\frac{dp}{dt}=\frac{mz_0}{l^2}q, \qquad \frac{dr}{dt}=-\frac{mx_0}{l}q$$

Comparing these linear equations with the first and third equations of system (1.3), we find the equality

$$\mu l^2\gamma_2=(Ax_0p+Cz_0r+\lambda l)\,q \tag{1.7}$$

which serves as a condition for their equivalence. We shall use this relation below. Thus, when equality (1.7) is observed, in what follows we examine a dynamic system of the form

$$\frac{dp}{dt}=\frac{mz_0}{l^2}q, \quad B\frac{dq}{dt}=(C-A)\,pr+\frac{\lambda}{l}(z_0p-x_0r)+ \tag{1.8}$$

$$\mu\,(x_0\gamma_3-z_0\gamma_1), \qquad \frac{dr}{dt}=-\frac{mx_0}{l^2}q$$

The quadratic integral

$$p^2+q^2+r^2=\text{const}=\omega^2 \tag{1.9}$$

must hold for regular precessions of the gyrostat. Using relation (1.6), we express q from (1.9) and we substitute it into the first equation of system (1.8). We obtain

$$\frac{dp}{dt}=\frac{m}{l^2}\,(z_0{}^2\omega^2-m^2+2mx_0p-l^2p^2)^{1/2}$$

An integration of this equation yields

$$p=\frac{n}{l}(x_0+z_0a\sin n\tau)$$

$$a=\frac{1}{m}\sqrt{l^2\omega^2-m^2}, \qquad n=\frac{m}{l}, \qquad \tau=t-t_0 \tag{1.10}$$

In the following we assume $a=1$. With due regard to (1.10), the angular velocity components q, r are found from relations (1.6), (1.9) without further integration.

Thus, we obtain the following solution of the system of equations (1.8), defining the gyrostat's angular velocity components (for $a=1$):

$$p=\frac{n}{l}(x_0+z_0\sin n\tau) \qquad q=n\cos n\tau \tag{1.11}$$

$$r=\frac{n}{l}(z_0-x_0\sin n\tau)$$

These formulas in no way differ in form from the corresponding formulas describing the regular precession of an absolutely rigid body [5] under conditions (1.1).

We pass on to the determination of the variables γ_1, γ_2, γ_3. The relation determining γ_2 already exists in the form of (1.7). Let us obtain the relations connecting γ_1, γ_3. For this we differentiate the integral (1.9) with respect to time; by virtue of system (1.7), we have

$$\frac{d}{dt}(p^2+q^2+r^2)=\frac{1}{Bl^2}qQ \tag{1.12}$$

$$Q=(Bm+l\lambda)\,(z_0p-x_0r)-(A-C)\,l^2pr+\mu l^2\,(x_0\gamma_3-z_0\gamma_1)$$

The right-hand side of equality (1.12) vanishes only for $Q=0$; the vanishing of variable q is impossible because of solution (1.11). To obtain a second relation connecting γ_1, γ_3, we make use of the energy integral

$$Ap^2+Bq^2+Cr^2+2\mu\,(x_0\gamma_1+z_0\gamma_3)=2h \tag{1.13}$$

We note that integral (1.13) clearly does not depend upon the gyrostatic terms occurring in the equations of system (1.3). By solving the system of algebraic equations $Q = 0$ and (1.13) with respect to γ_1, γ_3, we find

$$\gamma_1 = \frac{1}{\mu l^4}\left\{x_0\left[hl^2 - \frac{1}{2} l^2 (Ap^2 + Bq^2 + Cr^2)\right] + z_0\left[(Bm + \lambda l)(z_0 p - x_0 r) - (A - C)\, l^2 pr\right]\right\}$$

$$\gamma_3 = \frac{1}{\mu l^4}\left\{z_0\left[hl^2 - \frac{1}{2} l^2 (Ap^2 + Bq^2 + Cr^2)\right] - x_0\left[(Bm + \lambda l)(z_0 p - x_0 r) - (A - C)\, l^2 pr\right]\right\}$$

Substituting the solution (1.11) we have found into these formulas and into (1.7), after a number of reductions, we obtain

$$\gamma_1 = \frac{n^2}{\mu l^4}\left\{\frac{x_0}{2}\left(\frac{2h}{n^2} - (A + C)\right) l^2 + z_0 l^2\left(C + \frac{\lambda}{n}\right)\sin n\tau - (A - C)\, x_0 z_0^2 \cos^2 n\tau\right\} \qquad (1.14)$$

$$\gamma_2 = \frac{n^2}{\mu l^3}\left\{\left(H + \frac{\lambda}{n}\right) l^2 \cos n\tau + (A - C)\, x_0 z_0 \sin n\tau \cos n\tau\right\}$$

$$\gamma_3 = \frac{n^2}{\mu l^4}\left\{\frac{z_0}{2}\left(\frac{2h}{n^2} - (A + C)\right) l^2 - x_0 l^2\left(A + \frac{\lambda}{n}\right)\sin n\tau + (A - C)\, x_0^2 z_0 \cos^2 n\tau\right\}$$

From these formulas we see that gyrostatic terms depending on the absolute value of moment λ occur in their coefficients. Consequently, the dynamic characteristics on the internal cyclic motions in the gyrostat, influence the gyrostat's rotation about the vertical axis.

Let us refine formulas (1.14). To do this we substitute them into the kinematic equations of system (1.3). This substitution leads to the necessity of accepting that the total mechanical energy of the gyrostat

$$h = \tfrac{1}{2}(A + C)\, n^2$$

In final form solution (1.14) is written as

$$\gamma_1 = \frac{n^2}{\mu l^2}\left[\left(C + \frac{\lambda}{n}\right) z_0 \sin n\tau - (B - C)\, x_0 \cos^2 n\tau\right] \qquad (1.15)$$

$$\gamma_2 = \frac{n^3}{\mu l^3}\left[\left(H + \frac{\lambda}{n}\right) l^2 \cos n\tau + (A - C)\, x_0 z_0 \sin n\tau \cos n\tau\right]$$

$$\gamma_3 = \frac{n^2}{\mu l^2}\left[-\left(A + \frac{\lambda}{n}\right) x_0 \sin n\tau + (A - B)\, z_0 \cos^2 n\tau\right] \qquad (1.16)$$

$$H = A - B + C, \quad Hl^2 = Ax_0^2 + Cz_0^2$$

We make one more computation connected with the determination of the position of the gyrostat's fixed precession axis in space. If this axis is defined by the unit vector $\varkappa^\circ$ with components $(\varkappa_1, \varkappa_2, \varkappa_3)$ relative to the moving axes, then according to [6] the following relation must hold:

$$(\omega \cdot \varkappa^\circ) = \varkappa_1 p + \varkappa_2 q + \varkappa_3 r = \text{const} = s$$

The system of equations describing the rotation of the gyrostat relative to the fixed precession axis has the form

$$d\varkappa_1/dt = \varkappa_2 r - \varkappa_3 q, \quad \ldots \qquad (1, 2, 3) \quad (p, q, r)$$

Substituting formulas (1.11) here, we represent this system as

$$\frac{d\varkappa_1}{dt} = \frac{n}{l}(z_0 - x_0 \sin n\tau)\varkappa_2 - n\cos n\tau\varkappa_3 \tag{1.17}$$
$$\frac{d\varkappa_2}{dt} = \frac{n}{l}[(x_0 + z_0 \sin n\tau)\varkappa_3 - (z_0 - x_0 \sin n\tau)\varkappa_1]$$
$$\frac{d\varkappa_3}{dt} = n\cos n\tau - \frac{n}{l}(x_0 + z_0 \sin n\tau)\varkappa_2$$

Substituting formulas (1.11) into relation (1.16), we obtain

$$\frac{n}{l}\varkappa_1(x_0 + z_0 \sin n\tau) + n\varkappa_2 \cos n\tau + \frac{n}{l}\varkappa_3(z_0 - x_0 \sin n\tau) = s \tag{1.18}$$

Differentiating (1.18) with resoect to time, by virtue of system (1.17), we obtain

$$(z_0\varkappa_1 - x_0\varkappa_3)\cos n\tau - l\varkappa_2 \sin n\tau = 0 \tag{1.19}$$

Consequently, in order for relation (1.18) to be an integral of system (1.17), it is necessary that the solution of this system satisfy Eq. (1.19) as well.

Let us consider the system of equations, which is easily obtained from (1.18) and (1.19)

$$x_0\varkappa_1 + z_0\varkappa_3 = l\left(\frac{s}{n} - \frac{\varkappa_2}{\cos n\tau}\right)$$
$$z_0\varkappa_1 - x_0\varkappa_3 = l\frac{\sin n\tau}{\cos n\tau}\varkappa_2$$

Solving this system relative to $\varkappa_1$, $\varkappa_3$, we find

$$\varkappa_1 = \frac{1}{l}\left[\frac{s}{n}x_0 - \frac{x_0 - z_0 \sin n\tau}{\cos n\tau}\varkappa_2\right] \tag{1.20}$$
$$\varkappa_3 = \frac{1}{l}\left[\frac{s}{n}z_0 - \frac{z_0 + x_0 \sin n\tau}{\cos n\tau}\varkappa_2\right]$$

To eliminate $\varkappa_2$ from these formulas we make use of the trivial integral $\varkappa_1^2 + \varkappa_2^2 + \varkappa_3^2 = 1$ of system (1.17). Substituting (1.20) into this integral, we obtain an equation whose roots are

$$\varkappa_2 = \left(\frac{s}{2n} \pm \frac{1}{2}\sqrt{2 - \frac{s^2}{n^2}}\right)\cos n\tau \tag{1.21}$$

Replacing $\varkappa_2$ in formulas (1.20) by expressions (1.21), we have formulas for $\varkappa_1$, $\varkappa_2$, $\varkappa_3$, the substitution of which into system (1.17) shows that they form a solution of this system if and only if we retain the plus sign in front of the radical and set $s = n$. We finally obtain

$$\varkappa_1 = \frac{z_0}{l}\sin n\tau, \qquad \varkappa_2 = \cos n\tau, \qquad \varkappa_3 = -\frac{x_0}{l}\sin n\tau \tag{1.22}$$

Hence it follows that, just as in the classical case of precession of an absolutely rigid body [5], the gyrostat rotates relative to the fixed precession axis. Comparing formulas (1.22) with (1.15), we conclude that if the gyrostat's rotation relative to the vertical fixed axis depends on the gyrostatic terms, then its rotation relative to the fixed precession axis, not coinciding with the vertical, clearly does not depend on the terms indicated and is accomplished exactly as if parts performing cyclic motions were not contained inside the gyrostat.

Let us clarify certain kinematic characterisitcs of the motion being investigated. We consider the product of vectors (1.6). Substituting here the appropriate formulas from (1.11), we obtain

$$(\mathbf{l}\cdot\boldsymbol{\omega}) = l\omega_l = ln = \nu = m = \mathrm{const} \tag{1.23}$$

Hence we see that the constant n is the angular velocity of the natural rotation of the gyrostat, while v is the linear velocity of the gyrostat's center of mass in its precession motion around the fixed precession axis. It turns out that the magnitude of velocity n is an arbitrary quantity connected by a one-parameter relation with the gyrostatic moment λ. This relation is found by substituting formulas (1.15) into the trivial integral $\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1$ of system (1.3). We obtain

$$n^2\lambda^2 + 2Hn^3\lambda + (H^2 + E^2)\, n^4 = \mu^2 l^2$$

For the gyrostatic moment λ this equation yields

$$\lambda_{1,2} = -Hn \pm \sqrt{\frac{\mu^2 l^2}{n^2} - E^2 n^2} \qquad (1.24)$$

The roots are real when $\mu l \geqslant En^2$. It follows from (1.24) that the velocity of the gyrostat's natural rotation in formulas (1.11), (1.15) proves to be an arbitrary constant. This is not so in the classical case of regular precession [5].

Let us now return to the previously-adopted assumption $a = 1$; using this value in (1.10), we find $l^2\omega^2 = 2m^2$. Since $m = ln$, we have $\omega = \sqrt{2}m / l = \sqrt{2}n$.

We further determine the angular velocity of the gyrostat's rotation around the fixed precession axis. By virtue of relation (1.18) and of formulas (1.22), we have $s = n = \omega_l$ in accordance with formula (1.23). We determine the angle α between the axes of the natural rotation and of the precession of the gyrostat from the equality

$$(\mathbf{l}\cdot\varkappa^\circ) = \cos\alpha = x_0\varkappa_1 + z_0\varkappa_3 = 0$$

Hence we conclude that $\alpha = \pi / 2$. We determine further the angle β of the deviation of the gyrostat's fixed precession axis from the vertical using the equality

$$(\varkappa\cdot\zeta^\circ) = \cos\beta = \gamma_1\varkappa_1 + \gamma_2\varkappa_2 + \gamma_3\varkappa_3$$

Substituting formulas (1.15) and (1.22) here, we obtain

$$\cos\beta = \frac{n^2}{\mu l}\left(H + \frac{\lambda}{n}\right) \qquad (1.25)$$

Finding λ from this and comparing the expression obtained with (1.24), we have

$$n = \pm\sqrt{\frac{\mu l}{E}\sin\beta} \qquad (1.26)$$

From this formula we see that we cannot choose the deviation angle β to equal $0, \pi$, otherwise the velocity n becomes zero. For deviation angles β for which $\cos\beta > 0$ or $\cos\beta < 0$ (Fig. 2), the inequalities

$$H + \lambda/n > 0, \qquad H + \lambda/n < 0 \qquad (1.27)$$

respectively, should be fulfilled according to formula (1.25). Their fulfillment imposes a restriction on the choice of the sign in front of the radical in (1.26).

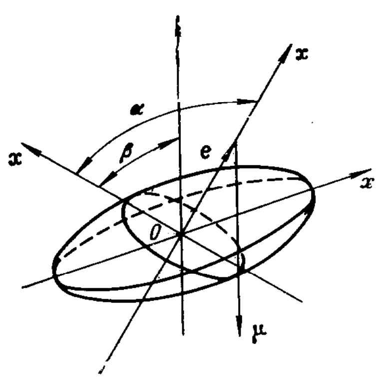

Fig. 2

With due regard to formula (1.26) we rewrite equality (1.25) as

$$\lambda / n = -H + E \operatorname{ctg} \beta$$

If $n > 0$, $\operatorname{ctg} \beta > 0$, $H < E \operatorname{ctg} \beta$, then $\lambda / n > 0$ and the first inequality of (1.27) is satisfied. If $n < 0$, $\operatorname{ctg} \beta > 0$, $H > E \operatorname{ctg} \beta$, then $\lambda / n < 0$ and the first inequality again is satisfied. The second inequality of (1.27) is satisfied if $n < 0$, $\operatorname{ctg} \beta < 0$, in which case $\lambda / n < 0$. Hence it follows that in the given case the cyclic motion in the gyrostat should be accomplished only in the direction opposite to the gyrostat's natural rotation. If, in particular, the deviation angle $\beta = \pi / 2$, the gyrostat's precession motion is performed around the horizontal axis. Here

$$\lambda = -Hn, \quad n = \pm \sqrt{\mu l / E} \tag{1.28}$$

follows from fornulas (1.25), (1.26). Thus, in this case cyclic motion in the gyrostat is possible both in the same as well as in the opposite direction as the gyrostat's natural rotation around the barycentric axis. With due regard to equalities (1.28), formulas (1.15) have the form

$$\gamma_1 = -\frac{h^2}{\mu l^2} [(A - B) z_0 \sin n\tau + (B - C) x_0 \cos^2 n\tau]$$

$$\gamma_2 = \frac{n^2}{\mu l^3} (A - C) x_0 z_0 \sin n\tau \cos n\tau$$

$$\gamma_3 = \frac{n^2}{\mu l^2} [-(B - C) x_0 \sin n\tau + (A - B) z_0 \cos^2 n\tau]$$

In such a motion of the gyrostat its center of gravity describes a circle in a vertical plane passing through the fixed point of the body. With due regard to (1.28) the linear velocity of this motion is

$$v = ln = \pm l \sqrt{\mu l / E}$$

At first glance this motion seems to be strange, since here the accelerating and decelerating actions of the force of gravity on the gyrostat are in fact excluded. We could show that such actions of the force of gravity on the gyrostat should be compensated by the gyroscopic moments as a consequence of the relative cyclic motions and of the natural transfer rotation of the gyrostat. These moments can be created only by the Coriolis forces of inertia and of translational motion. Therefore, the regular precession of the gyrostat proves to be of greater interest because of the possibility of controlling it. In fact, if this can happen for an absolutely rigid body [5] only for an angle $0 < \beta < \pi / 2$ which is determined by the equality

$$\cos \beta = \frac{n^2}{\mu l} H < 1$$

then it happens for the gyrostat for angles β determined by equality (1.25), i.e. within the limits $0 < \beta < \pi$. Here the function (1.24) is the controlling function.

The following result has emerged as a result of the analysis carried out: the form of the gyrostat's motion in space, determined by solutions (1.11), (1.22), turns out to be invariant just as in the classical case (Grioli).

2. We investigate the gyrostat's regular precession under the Lagrange conditions: $A = B \neq C$, $x_0 = y_0 = 0$, $z_0 = l$, $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = \lambda$. Then solutions (1.11), (1.15) and (1.22) can be written as

$$p = n \sin n\tau, \quad q = n \cos n\tau, \quad r = n = \text{const} \tag{2.1}$$
$$\gamma_1 = \frac{n}{\mu l}\left(C + \frac{\lambda}{n}\right)\sin n\tau, \quad \gamma_2 = \frac{n^2}{\mu l}\left(C + \frac{\lambda}{n}\right)\cos n\tau, \quad \gamma_3 = 0$$
$$\varkappa_1 = \sin n\tau, \quad \varkappa_2 = \cos n\tau, \quad \varkappa_3 = 0$$

where, in accordance with (1.24) and in view of $E = 0$,

$$\lambda = -Cn \pm \frac{\mu l}{n} \tag{2.2}$$

In the case under consideration the deviation angle β of the gyrostat's precession axis from the vertical is determined by the equality

$$\cos \beta = \gamma_1 x_2 + \gamma_2 \varkappa_2 = \frac{n^2}{\mu l}(C + \lambda / n)$$

Finding λ from this and comparing with (2.2), we are convinced that $\cos \beta = \pm 1$, $\beta = 0, \pi$. Consequently, the gyrostat's regular precession, described by formulas (2.1), can be accomplished only around the vertical axis. The equalities

$$\gamma_1 = \varkappa_1, \quad \gamma_2 = \varkappa_2, \quad \gamma_3 = \varkappa_3$$

are fulfilled in this case. From them, with due regard to formulas (2.1), we obtain the one equation

$$Cn^2 + \lambda n - \mu l = 0$$

revising the value of $\cos \beta$ up to unity. By solving this relative to velocity n, we obtain

$$n = \frac{-\lambda \pm \sqrt{\lambda^2 + 4C\mu l}}{2C} \tag{2.3}$$

When $\lambda = 0$ formulas (2.1), (2.3) describe regular precession in the classical case [5].

3. We investigate external forces capable of causing the regular precession of the gyrostat. It is well known [7] that a body fixed at one point and possessing an axis of kinetic symmetry, can accomplish regular precession if and only if the principal moment $\mathbf{L}_0$ of the external forces relative to the fixed point O is constant in absolute value and is directed along the nodal line. Let us ascertain whether this condition is satisfied in the case of a heavy asymmetric gyrostat. According to (1.3) we have

$$L_x = A\frac{dp}{dt} - (B - C)qr + \frac{z_0}{l}\lambda q$$
$$L_y = B\frac{dq}{dt} + (A - C)pr - \frac{\lambda}{l}(z_0 p - x_0 r)$$
$$L_z = C\frac{dr}{dt} - (A - B)pq - \frac{x_0}{l}\lambda q$$

Substituting here the values of p, q, r from (1.11), with due regard to (1.4), we obtain

$$L_x = \frac{n^2}{l} z_0 \left[\left(H + \frac{\lambda}{n}\right)\cos n\tau + E \sin n\tau \cos n\tau\right] \tag{3.1}$$
$$L_y = -n^2\left[\left(H + \frac{\lambda}{n}\right)\sin n\tau - E \cos^2 n\tau\right]$$
$$L_z = -\frac{n^2}{l} x_0 \left[\left(H + \frac{\lambda}{n}\right)\cos n\tau + E \sin n\tau \cos n\tau\right]$$

Using (3.1) we find the absolute value of the moment of the external forces

$$L_0 = \sqrt{L_x^2 + L_y^2 + L_z^2} = n^2 \sqrt{\left(H + \frac{\lambda}{n}\right)^2 + E^2 \cos^2 n\tau}$$

As we see, the moment L_0 is not constant. But under the Lagrange conditions it does prove to be constant. The principal moment L_0 should lie in the horizontal plane since it is created by a unique force, namely, the force of gravity applied to the body

$$(L_0 \cdot \zeta^\circ) = L_x\gamma_1 + L_y\gamma_2 + L_z\gamma_3 = 0$$

As a consequence of the gyrostat's rotation around the fixed precession axis with velocity $n = \omega_l$, the principal moment L_0 precesses in the horizontal plane with the velocity

$$\omega_1 \cos\beta = \frac{n^2}{\mu l}\left(H + \frac{\lambda}{n}\right).$$

According to [7], for the principal moment L_0 to coincide with the nodal line it is necessary that the projections L_x, L_y, L_z be expressed in terms of the Euler angles in the following manner:

$$L_x = L_0 \cos\varphi, \qquad L_y = -L_0 \sin\varphi, \qquad L_z = L_0 \cos\frac{\pi}{2} = 0$$

According to (3.1), $L_z \neq 0$. Consequently, in the given case the principal moment is not located along the nodal line in the horizontal plane. But, in the case of gyrostat precession under the Lagrange conditions this requirement is fulfilled since $L_z = 0$.

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